DOCUMENT RESUME

ED 270 504 TM 860 394

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TITLE Grade 10 Students' Understanding of Basic Algebraic Concepts.
PUB DATE 18 Apr 86
NOTE 24p.; Paper presented at the Annual Meeting of the American Educational Research Association (70th, San Francisco, CA, April 16-20, 1986).
PUB TYPE Speeches/Conference Papers (150) -- Reports - Research/Technical (143) -- Tests/Evaluation Instruments (160)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Algebra; Comparative Analysis; Concept Formation; Cross Cultural Studies; Educational Assessment; Foreign Countries; Grade 10; High Schools; *Mathematical Concepts; *Mathematics Achievement; *Mathematics Tests; Multiple Choice Tests; *Problem Solving; Student Evaluation; Test Items; Test Reliability
IDENTIFIERS *British Columbia; *England

ABSTRACT

One of the most ambitious attempts to assess students' understanding of mathematics was carried out in England between 1974 and 1979. The results of this study indicate that there is a need in an overall assessment program to evaluate students' algebraic understanding as well as to measure their achievement in algebraic manipulations and problem solving. The present study is a modified replication of the English study. In 1977, 1981, and 1985 the British Columbia Ministry of Education carried out large scale assessments of mathematics achievement. Questions were designed to assess students' levels of algebraic understanding ranging from basic simplification to difficult relationships. The responses were classified by coders into one of nine categories, for example, correct, correct but weak, ambiguous, letter ignored. This scheme was extended further by coding the specific responses. It was thus possible to obtain frequency counts of each individual response as well as to assess overall error rates. A reliability check on coding yielded a 95.8 percent agreement rate on the 425 returned booklets. Four criteria were used to determine sample representativeness: gender, type of mathematics course, future mathematics plans, and performance on three multiple-choice items. Results indicate that performance of students on the three multiple-choice items in the sample was significantly higher than the population. Possible causes for this result are discussed. The appendix contains the British Columbia Mathematics Assessment (1985) for grade 10. (PN)

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# Grade 10 Students' Understanding of Basic Algebraic Concepts

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Paper presented at the 1986 Annual Meeting of the

American Educational Research Association

San Francisco, CA

April 18, 1986

TM 860 394

In recent years a number of North American researchers have investigated students' understanding of various algebraic concepts. Herscovics and Kieran (1980) focussed on the general notion of equations. Rosnick (1981) examined the responses of students who were asked to create one very specific equation. Wagner (1983) has summarized for teachers much of the research concerning students' understanding of variables. In a clinical study, Wagner, Rachlin, and Jensen (1984) investigated how students learn mathematical content (integers, fractions, and polynomials) and the process by which it was learned (generalizability and reversibility). These studies have focussed on single concepts or were based on small number of subjects.

One of the most ambitious attempts to assess students' understanding of mathematics was carried out in England between 1974 and 1979 (see Hart, 1981). Written tests were administered to over 10 000 students and each student in the sample was tested on at least two of ten topics. The results of this study, as well as those of other investigations of how children learn mathematics, have been synthesized for teachers in Dickson, Brown, and Gibson (1984).

Part of the Hart study examined 13- to 15-year-old children's perceptions of secondary school algebra, or generalized arithmetic. From their results the researchers hypothesized six different ways in which students interpret and use letters in algebra. These are, in hierarchic order, proceeding from the simplest: letter evaluated, letter not used, letter used as object, letter used as specific unknown, letter used as generalized number, and letters used as variables. The research team also outlined their notion of "structural complexity" as an important determinant of the difficulty of an item. The combination of these two variables--interpretation and complexity--enabled them to classify items into four levels of understanding. The authors concluded "on the algebra test the majority of 13, 14 and 15 year olds were at Levels 1 and 2 (73, 59 and 53 per cent respectively) and were not able to cope consistently with items that can properly be called algebra at all, i.e., items where the use of letters as unknown numbers cannot be avoided" (Hart, 1981, p. 118).

The results of Hart's study indicate that there is a need in an overall assessment program to evaluate students' algebraic understanding as well as to measure their achievement in algebraic manipulations and problem solving. The study reported here is a modified replication of the English study which was carried out in a Canadian Province.

## Method

In 1977, 1981, and 1985 the British Columbia Ministry of Education carried out large scale assessments of mathematics achievement. In the first two assessments, multiple-choice items only were used to determine performance of students in Grades 4, 8 and 12 on a comprehensive set of objectives, most of which were specifically related to the centrally-set curriculum. In 1985, the team which was contracted to develop the assessment instruments and report on achievement for Grades 4, 7, and 10 included, for the first time, two open-ended instruments at each grade level, in addition to the usual multiple-choice forms.

In May 1985, each student in Grades 4, 7, and 10 responded to one of the assessment instruments. Materials sent to each classroom in the province included one copy of one of the two open-ended instruments randomly embedded among the standard multiple-choice forms. Each classroom teacher was instructed to hand out the forms in the order received, thereby generating a random selection of students who responded to the open-ended forms. The population consisted of approximately 25 000 to 30 000 students, and 1000 to 1200 classrooms, at each grade level. As a result, data from each open-ended instrument was obtained from about 400 to 500 students at each grade level.

Booklets were returned to the contract team and three markers were hired to code the results. Each booklet required a unique coding system which was developed from theoretical considerations and from an inspection of a 10 percent sample of the booklets. The coders also identified items which yielded unusual, original, or otherwise interesting responses. A reliability check was carried out on a 7 percent sample for each form. The responses were keypunched and analyzed using the SPSS computer package.

## Instrument

The instrument for which results are reported in this paper was a slightly modified version of the "Algebra 1" test paper reported in Johnson (1980, pp. 174-177). Questions were designed to assess students' levels of understanding of algebra, and ranged from basic simplification such as "2a + 5b + a =____" to difficult relationships such as "If the equation $(x + 1)^3 + x = 349$ is true when $x = 6$, then what value of x will make the following equation true? $(5x + 1)^3 + 5x = 349$ ." Other items required students to do such things as make a

statement (e.g., "What can you say about m if m = 3n + 1 and n = 4?"), make necessary substitutions (e.g., "If e + f = 8, then e + f + g = ____"), and explain relationships (e.g., "Which one is larger: 2n or n + 2? Explain your answer.") A copy of the instrument and selected results are included with this paper.

The coding scheme set out in Johnson (1980, pp. 178-180) was used as the basis for coding the results. In this system the responses were classified by the coders into one of nine categories, for example, correct, correct but weak, ambiguous, letter ignored. The Johnson scheme was extended further by coding the specific responses. In this was it was possible to obtain frequency counts of each individual response as well as to assess overall error category rates.

Several multiple-choice items which appeared on the standard instruments were also included on the open-ended forms. These items, and items at the Grade 10 level which asked students information about the class in which they were enrolled and their plans for taking mathematics in the following year, were included to permit an objective assessment of the representativeness of the sample.

## Results

A total of 425 booklets were returned and all of these contained useable data. Inspection of the raw data file indicated that virtually all students attempted most of the items.

The reliability check on the coding yielded a rate of agreement between the two coders of 95.8%. Discussion of the discrepant values showed about half to be a result of coding slips and half to be real disagreements as to the proper interpretation.

To determine the representativeness of the sample, four criteria were used: gender, type of mathematics course enrolled in, plans for taking mathematics next year, and performance on three multiple-choice items. The results showed the same even split between males and females as for the population. However, 83% of students in the sample indicated that their mathematics course was designed to prepare them for Grade 11 Algebra (the academic mathematics course), in comparison to 73% for the population. Furthermore, 76% of the sample listed Algebra 11 as the course they planned to take in Grade 11 as compared to 62% in the population of Grade 10 students. Finally,

performance of students in the sample was significantly higher than the population on the three multiple-choice items (See attached results).

These results indicate that the obtained sample of students was superior in mathematical ability to the population. Better performance on the multiple-choice items might be accounted for because students selected to complete the open-ended instrument did not have to complete demographic and attitudinal items as did the population of students, and thus had more time to complete the test items. However, the higher proportion of students who planned to take an academic mathematics course in Grade 11 suggests that some teachers selected students in their class to write the one different form. Moreover, the higher proportion of students who indicated that they were enrolled in an academic mathematics class may indicate some biased selection of students at the school level. As a result, it is likely that the results reported here are from a sample of students who are somewhat higher in mathematics ability than the population of Grade 10 students.

Comparison with British students

Kuchemann (1978) reported results from the British study for students in the third year of secondary school. Grade 10 students in British Columbia are also in their third year of secondary school (B.C. is an anomaly among Canadian provinces). The average age of British students was approximately 14.3 years, and that of B.C. students was about 15.1 years. The British results were based on approximately 1000 students drawn from 12 comprehensive and 3 selective schools, selected so that their IQ distribution would be representative of that in English schools. In comparing results from the two jurisdictions it must be borne in mind, therefore, that the B.C. students were tested eight years later than their English counterparts, they were slightly older, and the B.C. sample was likely biased toward higher ability. Assuming similar curricula, one might assume B.C. students to perform better than the British students.

This was indeed the case. Kuchemann (1978) reported results on a selection of 26 of the 50 items in the British study. His table, with the results from B.C. inserted, is shown in Figure 1. "Item" refers to the designation used in Kuchemann's article; "Question Number on Test" refers to the numbering used on the B.C. instrument.

| Piagetian Sub-Stage | Item | Question Number on test | % Correct B.C. | % Correct | EVALUATED | IGNORED | OBJECT | SPECIFIC UNK | GNRLSD NMBR | VARIABLE | QUESTION |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Early Concrete | A | 8a | 96 | 97 | | x | | | | | If $a+b=43$, $a+b+2=....$ |
| | B | 12a | 97 | 94 | | | x | | | | $p=....$ |
| | C | 9a | 96 | 92 | x | | | | | | If $a+5=8$, $a=....$ |
| | D | 10a | 89 | 91 | | | | | | | $A=....$ |
| | E | 10b | 89 | 89 | | | | | | | $A=....$ |
| | F | — | — | 88 | | | | | | | If $x-x+2$, $6-....$ |
| Late Concrete | G | 8b | 84 | 74 | | x | | | | | If $n-246=762$, $n-247=....$ |
| | H | 4 | 89 | 72 | | x | | | | | smallest, largest of: $n+1$, $n+4$, $n-3$, $n$, $n-7$ |
| | I | 6a | 71 | 68 | | x | | | | | Add 4 onto $n+5$ |
| | J | 10c | 88 | 68 | | | x | | | | $A=....$ |
| | K | 12b | 81 | 68 | | | x | | | | $p=....$ |
| | L | 12c | 79 | 64 | | | x | | | | $p=....$ |
| | M | 13b | 92 | 62 | x | | | | | | If $m=3n+1$ and $n=4$, $m=....$ |
| | N | 13a | 94 | 61 | x | | | | | | If $u=v+3$ and $v=1$, $u=....$ |
| Early Formal | O | 8c | 66 | 41 | | | | x | | | If $e+f=8$, $e+f+g=....$ |
| | P | 16 | 52 | 41 | | | | x | | | If $r=s+t$ and $r+s+t=30$, $r=....$ |
| | Q | 12d | 65 | 38 | | | | x | | | $n$-sided polygon, each side of length 2; $p=....$ |
| | R | 6b | 77 | 36 | | | | x | | | Add 4 onto $3n$ |
| | S | 18 | 43 | 30 | | | | | x | | What can you say about $c$ if $c+d=10$ and $c$ is less than $d$ |
| | T | 20b | 41 | 25 | | | | | x | | Is $L+M+N=L+P+N$ always, sometimes or never true? |
| | U | 22 | 47 | 22 | | | | x | | | Cakes cost $c$ pence each and buns cost $b$ pence each. If I buy 4 cakes and 3 buns, what does $4c+3b$ stand for? |
| Late Formal | V | 7a | 45 | 17 | | | | x | | | Multiply $n+5$ by 4. |
| | W | 10d | 43 | 12 | | | | x | | | $A=....$ |
| | X | 24 | 55 | 11 | | | | x | | | "blue and red pencils" (see text). |
| | Y | 19 | 27 | 5 | | | | x | | | Mary's basic wage is £20 per week. She is also paid another £2 for each hour of overtime that she works. If $h$ stands for the number of hours of overtime that she works, and if $W$ stands for her *total* wage (in £'s) write down an equation connecting $W$ and $h$: ..... |
| | Z | 5 | 24 | 6 | | | | | | x | Which is larger, $2n$ or $n+2$? Explain. |

Figure 1. Selected item results. (Source: Kuchemann, 1978)



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The mean p-values for each group of questions are shown in Table 1.

Table 1
Mean p-values by groups of items

| Item Group | No. of Items | B.C. Students | British Students | Difference |
|---|---|---|---|---|
| EC | 5 | 93.4 | 92.6 | 0.8 |
| LC | 8 | 84.8 | 67.1 | 17.7 |
| EF | 7 | 55.9 | 33.3 | 22.6 |
| LF | 5 | 38.8 | 10.2 | 28.6 |

B.C. students' performance was better in each of the four groups of items. Differences for the Early Concrete group were minimal due to a ceiling effect. The difference increased with each succeeding group. For the Late Formal group, almost four times as many B.C. students were successful.

It might be argued that, because the item groupings were based on Piagetian levels, the achievement differences can be attributed to the age difference which favours B.C. by about one year. Johnson (1980) provided details of the British test results for each of the three age levels tested in 1976. For the 15.3 year age level the mean p-values for the groups of items listed in Kuchemann were 92.2, 70.8, 37.6, and 14.4. Thus, there was virtually no change in performance on the first group of items, and a gain of approximately four percentage points in each of the other three groups. Achievement levels for British students of age 15.3 years are still substantially less than those of the B.C. students.

While the difference in achievement levels may be of concern to those interested in international horse races, the relative difficulties of the items within the two groups provide some measure of the commonality of students' difficulties. The Pearson correlation between percent correct values from the two jurisdictions on the items listed in Figure 1 was 0.92. The correlation between performance of B.C. students and British 15.3-year-olds was 0.93.

Thus, while differences in curriculum might account for achievement differences, there seems to be a more general

common factor operating to produce similar relative achievement among items. Rather than attributing performance to Piagetian level, it may be more useful to look at the six categories of meaning that letters used in algebraic formulations may have for students. (For a full discussion of these categories, see Hart, 1981, Chap. 8). Each item in Figure 1 has been so classified, and the mean p-values for the categories are shown in Table.2.

Table 2
Mean p-values by category of meaning

| Category | Number of Items | B.C. Students | British Students |
|---|---|---|---|
| Letter evaluated | 3 | 94 | 72 |
| Letter ignored | 4 | 85 | 78 |
| Letter as object | 4 | 86 | 74 |
| Specific unknown | 9 | 53 | 25 |
| Generalized number | 2 | 42 | 28 |
| Variable | 1 | 24 | 6 |

The mean p-values for the categories indicate increasing difficulty, although the last two categories contain too few items to generalize. A more meaningful grouping of items may be into just two categories: (1) letter evaluated, ignored, or used as object, all of which are ways of avoiding generalized arithmetic, that is, by not using the letters as genuine unknowns (Hart, 1981, p. 108), and (2) letters used in some sense as algebraic entities, even though, as Hart points out, "the idea of a specific unknown number is still a rather primitive notion" (p. 108). Students in both B.C. and Britain performed much better on items in the former category than the latter. This may well reflect the tendency to teach the mechanics of algebra rather than the concepts.

Analysis within the B.C. sample

After the general level of performance was assessed, the response patterns to each item were analyzed for gender differences. No item yielded significant differences.

To assess the specific question of whether students interpreted letters as true variables, I looked more carefully at results on six particular items. These items, and the responses accepted as correct for this analysis, together with the proportion of the sample giving the response, are shown in Table 3.

Table 3

Items related to the concept of letter as variable

| Item No. | Item | Response | Percent |
|---|---|---|---|
| 5 | Which one is larger: 2n or n+2? | "depends"..at least one of 2, 1, 0 | 18 |
| | | "depends"..only one of 2, 1, 0 | 6 |
| | | "usually 2n" | 4 |
| 9b | What can you say about b if b+2 is equal to 2b? | b=2 | 56 |
| 16 | What can you say about r if r=s+t and r+s+t=30? | r=15 | 52 |
| 21a | a=b+3. What happens to a if b is increased by 2? | a increases by 2 | 60 |
| 21b | f=3g+1. What happens to f if g is increased by 2? | f increases by 6 | 25 |
| 20b | L+M+N=L+P+N. True always? Never? Sometimes, when___. | sometimes... | 48 |

Three of the six items, 9b, 21a, and 21b, were not discussed in either Kuchemann (1978) or Hart (1981). In Item 9b, the letter is evaluated but the relation is complicated by the use of the letter on both sides of the equation. Items 21a and 21b required the students to treat the letters at least as generalized numbers, particularly in the case of 21b.

On each of the items used here, the students were invited to express an opinion rather than to supply the correct answer. In this way much more latitude was given to the student to respond in a way which best represented his or her understanding of the problem. This collection of items tapped the student's understanding of letters and relationships rather than the ability to manipulate algebraic symbols. A student who answered all six items satifactorily might be thought of as algebraically sophisticated. The proportions of students who indicated their gender (3% did not) and correctly answered each set of items is shown in Table 4.

Table 4
Percent correct on "algebraic" items

| Item set | Females(N=197) n | % | Males(N=197) n | % | Total(N=394) n | % |
|---|---|---|---|---|---|---|
| 5 | 48 | 24 | 73 | 37 | 121 | 31 |
| 5&9b | 36 | 18 | 57 | 29 | 93 | 24 |
| 5&9b&16 | 21 | 11 | 46 | 23 | 67 | 17 |
| 5&9b&16&21a | 16 | 8 | 37 | 19 | 53 | 13 |
| 5&9b&16&21a&21b | 9 | 5 | 26 | 13 | 34 | 9 |
| 5&9b&16&21a&21b&20a | 6 | 3 | 19 | 10 | 25 | 6 |

From this analysis, something like six percent of the Grade 10 students in B.C. might be thought of as algebraically literate. Moreover, there appear to be consistent and significant sex differences, with perhaps twice as many males demonstrating understanding at each step of the analysis.

## Discussion

One of the most interesting items on the instrument was 20b: "f=3g+1. What happens to f if g is increased by 2?" A quarter of the students gave the correct answer that f increases by 6. Nine percent indicated that f would increase, but they did not attach a value to the increase. Many others, 28%, stated that f would increase by an amount other than 6, usually 2 or 3. A number of students (7%) substituted the value of 2 for g and evaluated f as 7. Others showed an unsuccessful algebraic formulation, for example, by increasing the coefficient of g: f=5g+1 (4%), or by increasing the constant by 2: f=3g+3 (4%). Others attempted to manipulate the equation, for example, f+2=3g+3 (1%), and f+2=3(g+2)+1 (1%). Some were unable to deal with the idea of g as a variable, and formulated a new expression as if g had not changed, for example, f=3g+7 (3%). Some (3%) felt that f should decrease.

Item 20a ("a=b+3. What happens to a if b is increased by 2?") was similar to Item 20b, but less complex. Sixty percent of the students were correct, but a wide variety of responses showed a lack of understanding by many students of such a simple relationship. Again, the critical factor appeared to be the inability of students to conceive of the letter as anything but a symbol representing some unknown but fixed value, that is, as "the answer" to an equation.

In both items a functional relationship exists between the two variables. This may be a fruitful area for further investigation, perhaps particularly so if students' understanding of the graphical representation of such relationships is examined.

In general, B.C. students did well compared to British students. Better performance on some items may be just a matter of curriculum. For example, B.C. students did much better on items such as "If m=3n+1 and n=4, then m=...", where a simple substitution was required. Performance on other items, however, may be less attributable to teaching, for example, "Which is larger: 2n or n+2?" or "If e+f=8, e+f+g=...". If do not believe such items are a standard part of the B. C., or any other North American, curriculum.

In 1981, the most popular textbook series in B.C. for Grades 1 through 6 was Investigating School Mathematics (Addison-Wesley). In Grades 7 and 8, the two most widely used textbook series were Mathematics I and II (Ginn) and School Mathematics I and II (Addison-Wesley). In Grades 9 and 10, about sixty percent of teachers used Mathematics for a Modern World (Books 1 and 2) (Gage) and about thirty percent used Modern Algebra (Book 1) (Houghton Mifflin). These textbooks are predominantly authored by Americans and reflect an American approach to teaching algebra. Recent results from the Second International Mathematics Study indicate minimal differences in content, attitudes of students, and teaching methods between British Columbia and the U.S.A. (see, for example, Rogers & O'Shea, in press).

The notion seems to be pervasive in North America that other jurisdictions teach mathematics better than we do. Achievement by Japanese students is legendary. They are today what the Russians were 30 years ago. Nevertheless, perhaps it is time to assess our strengths as well as our weaknesses. Recently, two British mathematics educators commented extensively on North American mathematics education (Lingard, Johnson, & O'Brien, 1985). From their remarks, one would assume that our pedagogy is compartmentalized and superficial, while the British approach is integrated and stresses mathematical understanding. The results of the present study should help to dispell some of the pessimism one might feel on reading such accounts.

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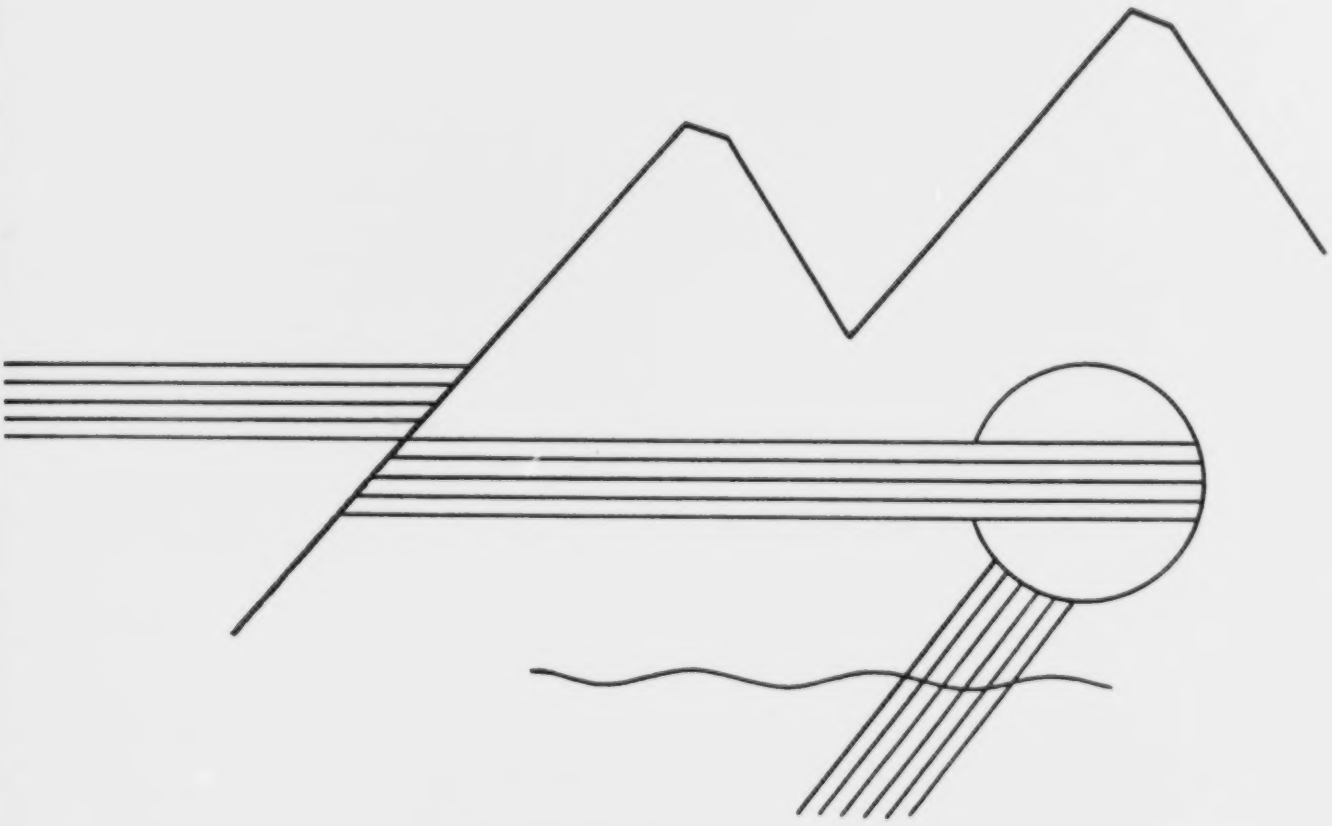
BRITISH COLUMBIA

# MATHEMATICS ASSESSMENT 1985

**GRADE 10**

# Q-1



## PART A - BACKGROUND QUESTIONS
## GRADE 10

1. Are you a boy or a girl? Check the box that applies.

GIRL ☐ 49% BOY ☐ 49% Missing 3%

2. What do you think are the most important reasons for studying mathematics?

| | First | Second | Third |
|---|---|---|---|
| job oriented | 27% | 11% | 1% |
| life skills | 30 | 21 | 3 |
| pre-req further study | 8 | 5 | 2 |
| logic/prob solve/brain | 9 | 5 | 2 |
| like/fun/aesthetics | 1 | - | - |
| other | 18 | 6 | 1 |
| missing | 7 | 53 | 92 |

3. Was the Mathematics 10 course you took earlier this year or are currently taking designed to prepare you to take Algebra 11?

| | |
|---|---|
| yes | 82% (Pop=73%) |
| no | 11 |
| not sure | 3 |
| other | 4 |

4. Which mathematics course do you plan to take in Grade 11?

| | |
|---|---|
| Algebra 11 | 75% (Pop=62%) |
| Mathematics 11 | 2 |
| Consumer Math 11 | 9 |
| Trades Math 11 | 6 |
| Mathematics 10 | 1 |
| Other | 7 |

5. What do you plan to do after leaving secondary school?

| | |
|---|---|
| University | 46% |
| Community College | 22 |
| Bus Sch/Tech Inst | 3 |
| Voc/Art/Trade/Train | 12 |
| Work | 7 |
| Travel | 1 |
| Don't know | 6 |
| Other | 4 |

## PART B - ACHIEVEMENT SURVEY
## GRADE 10

CIRCLE THE CORRECT RESPONSE IN QUESTIONS 1 TO 3.

1. There are 13 boys and 15 girls in a group. What fraction of the group is boys?

| % Response | | | |
|---|---|---|---|
| 9 | | A. | $\frac{15}{28}$ |
| 46 | | B. | $\frac{13}{15}$ |
| 3 | | C. | $\frac{15}{13}$ |
| 41 | (Pop=35%) | D. | $\frac{13}{28}$ |
| 1 | | E. | I don't know. |

2. The closest estimate for $\sqrt{640}$ would be

| | | | |
|---|---|---|---|
| 8 | | A. | 20 |
| 3 | | B. | 30 |
| 55 | (Pop=39%) | C. | 25 |
| 30 | | D. | 80 |
| 4 | | E. | I don't know. |

3. A used automobile can be bought for cash for $2850, or on credit with a downpayment of $400 and $80 a month for three years. How much more would a person pay by buying on credit than by buying the car for cash?

| | | | |
|---|---|---|---|
| 12 | | A. | $3280 |
| 6 | | B. | $ 640 |
| 71 | (Pop=55%) | C. | $ 430 |
| 4 | | D. | $ 400 |
| 7 | | E. | I don't know. |

SHOW ALL OF YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS IN THIS BOOKLET.

4. Which one is the smallest of these? _____

| | | | | |
|---|---|---|---|---|
| 2% | 1% | 1% | 9% | 86% |
| n + 1 | n + 4 | n - 3 | n | n - 7 |
| 1% | 92% | 0% | 1% | 4% |

Which one is the largest? _____

5. Which one is larger: 61% 2n or 9% n + 2? _____

Explain your answer.

| | | |
|---|---|---|
| "Depends" : ... | at least two of 2, 1, 0 | 18% |
| ... | only one of 2, 1, 0 | 6% |
| "Usually 2n" | with no examples | 4% |

6. 4 added to n can be written as n + 4.

Add 4 onto each of these:

| 8 | n + 5 | | 3n | |
|---|---|---|---|---|
| | n+9 | 71% | 3n+4 | 77% |
| | n+5+4 | 15% | 3n+4 | 4% |
| | 9 | 8% | 7n | 6% |
| | | | 12 | 4% |
| | | | 7 | 2% |

7. n multiplied by 4 can be written as 4n.

Multiply each of these by 4.

| 8 | n + 5 | | 3n | |
|---|---|---|---|---|
| | 4(n+5) | 17% | 12n | 64% |
| | 4n+20 | 45% | 3nx4 | 10% |
| | 4n+5 | 13% | 4(3n) | 11% |
| | n+20 | 8% | 12 | 8% |
| | 20n | 4% | 7n | 2% |
| | 20 | 3% | (4n)(3n) | 2% |
| | 9 | 2% | | |

8. If a + b = 43, then a + b + 2 =

| | |
|---|---|
| 45 | 96% |
| 41 | 1% |

If n - 246 = 762, then n - 247 =

| | |
|---|---|
| 761 | 84% |
| 763 | 3% |
| 269 | 3% |
| 762 | 2% |

If e + f = 8, then e + f + g =

| | |
|---|---|
| 8+g | 66% |
| 8g | 3% |
| 12 | 8% |
| 9 | 4% |
| 16 | 3% |
| 15 | 2% |
| 8 | 1% |

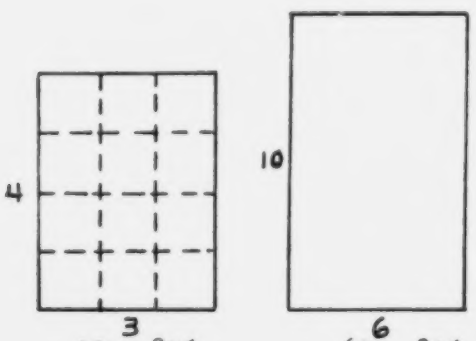
9. What can you say about a if a + 5 = 8?

| | |
|---|---|
| a = 3 | 96% |
| a = 8-5 | 1% |

What can you say about b if b + 2 is equal to 2b?

| | |
|---|---|
| b=2 | 56% |
| b+2=2b | 5% |
| b=2b-2 | 2% |
| b is 2 times bigger | 2% |
| b=1 | 3% |
| b=0 | 3% |
| b=b | 3% |
| b=unknown | 1% |
| impossible | 2% |

10. What are the areas of these shapes?



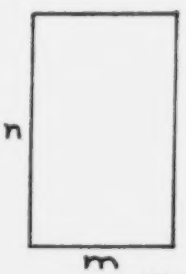

| | |
|---|---|
| 12 | 89% |
| 14 | 5% |
| 25 | 2% |
| 7 | 2% |

A = _____



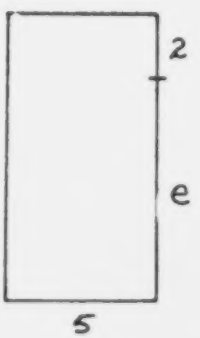

| | |
|---|---|
| 60 | 89% |
| 32 | 6% |
| 136 | 1% |
| 16 | 1% |

A = _____




| | |
|---|---|
| nm | 88% |
| 2n+2m | 3% |
| $n^2+m^2$ | 2% |
| n+m | 1% |

A = _____

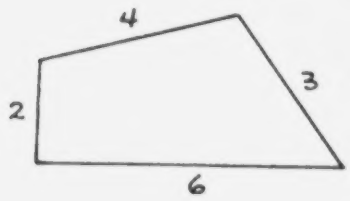


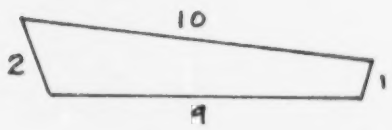

| | |
|---|---|
| 5(e+2) or equiv | 43% |
| 10e | 23% |
| 5e2 | 4% |

A = _____

11. The perimeter of this shape is equal to 6 + 3 + 4 + 2, which equals 15.

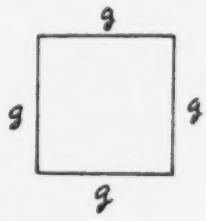
Determine the perimeter of this shape.



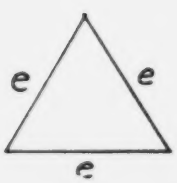




Perimeter = 22 98%

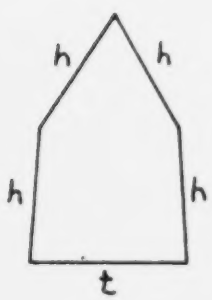
12. This square has sides of length g. So for its perimeter, we can write p = 4g



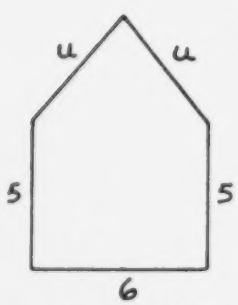

What can we write for the perimeter of each of these shapes?




P = 3e 97%

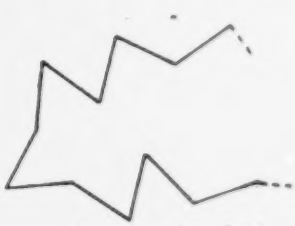



P = 4h+t 81%
4ht 12%
5h 2%




P = 2u+16 64%
(weak) 15%
2u16 2%
18u 2%
150x2u 2%
2u+31 3%

Part of this figure is not drawn. There are n sides altogether, all of length 2.



P = 2n or n2 65%
22n 2%
11n 3%
22 3%
n 4%

13. What can you say about u if u = v + 3 and v = 1?

| | |
|---|---|
| u=4 | 94% |
| u=-2 | 2% |

What can you say about m if m = 3n + 1 and n = 4?

| | |
|---|---|
| m=13 | 92% |
| m=14 | 1% |

14. If David has D records and Wendy has T records, what could you write for the number of records they have altogether?

| | |
|---|---|
| D+T | 85% |
| DT | 9% |
| DxT | 2% |

15. <u>a + 3a</u> can be written more simply as <u>4a</u>.

Write each of these more simply, where possible.

**2a + 5** =

| | |
|---|---|
| 2a+5 | 71% |
| 7a | 22% |
| 10a | 2% |

**2a + 5b** =

| | |
|---|---|
| 2a+5b | 77% |
| 7ab | 14% |
| 10ab | 1% |

**(a + b) + a** =

| | |
|---|---|
| 2a+b | 77% |
| $a^2$+ab | 5% |
| $a^2$+b | 5% |
| 2ab | 4% |

**2a + 5b + a** =

| | |
|---|---|
| 3a+5b | 85% |
| 8ab | 3% |

**(a + b) + (a - b)** =

| | |
|---|---|
| 2a | 53% |
| $a^2-b^2$ | 9% |
| 2a-2b | 7% |
| $a^2+b^2$ | 2% |
| 0 | 4% |

**3a - (b + a)** =

| | |
|---|---|
| 2a-b | 45% |
| 4a-b | 16% |
| 3a-(b+a) | 6% |
| 2a+b | 4% |

**a + 4 + a - 4** =

| | |
|---|---|
| 2a | 82% |

$a^2$ 3%
a 2%

**3a - b + a** =

| | |
|---|---|
| 4a-b | 80% |
| 2a-b | 5% |

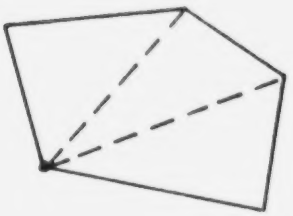
**(a - b) + b** =

| | |
|---|---|
| a | 60% |
| a-2b | 10% |
| (a-b)+b | 5% |
| $ab-b^2$ | 4% |
| a-bb | 3% |

16. What can you say about r if r = s + t and r + s + t = 30?

| | |
|---|---|
| r=15 | 52% |
| r=10 | 8% |
| r=30-s-t | 5% |
| r < 30 | 2% |
| r=30-s+t | 2% |

17. In a shape like this you can work out the number of diagonals by <u>taking away 3</u> from the number of sides.



So, a shape with 5 sides has 2 diagonals;

a shape with 57 sides has 54 84% diagonals;

a shape with k sides has k-3 75% diagonals.

k 6%

L, or any other letter 5%

-3k 1%

18. What can you say about c if c + d = 10 and c is less than d?

| | |
|---|---|
| systematic list | 24% |
| c < 5 | 19% |
| one value only | 16% |
| c < d | 10% |
| c ≤ 4 | 6% |
| c=10-d | 5% |

19. Tom's basic wage is $200 per week.
He is also paid another $10 for each hour of overtime that he works.

If h stands for the number of hours of overtime that he works, and if W stands for his <u>total</u> weekly wage (in dollars), write down an equation connecting W and h.

| | |
|---|---|
| W=200+10h | 27% |
| W+h | 17% |
| W=200+h | 11% |
| W+10h=wages | 8% |
| W+10h | 4% |
| 200W+10h | 4% |
| 200=W+h | 2% |
| W=W+h | 2% |

What would Tom's total wage be if he worked 4 hours of overtime?

| | |
|---|---|
| $240 | 86% |
| $40 | 5% |

20. When are the following true--always, never, or sometimes. (Underline the correct answer.)

A + B + C = C + A + B

Always 91% Never 1% Sometimes, when 5% _____

L + M + N = L + P + N

Always 3% Never 46% Sometimes, when _____

Sometimes. 7%

Sometimes, when M=P 41%

21. **a = b + 3. What happens to a if b is increased by 2?**

| | |
|---|---|
| a increases by 2 | 60% |
| a=5 | 8% |
| a=b+5 | 7% |
| a increases | 5% |
| a=2b+3 | 3% |
| a+2=b+5 | 2% |
| a+2=b+3 | 2% |
| a+2 | 3% |
| decreases | 2% |

**f = 3g + 1. What happens to f if g is increased by 2?**

| | |
|---|---|
| f increases by 6 | 25% |
| f increases by some amount | 28% |
| f increases | 9% |
| f=5g+1 | 4% |
| f=3g+3 | 4% |
| f=3(g+2)+1=3g+7 | 3% |
| f=7 | 7% |
| decreases | 2% |

22. **Muffins cost m cents each and buns cost b cents each. If I buy 4 muffins and 3 buns, what does 4m + 3b stand for?**

| | |
|---|---|
| total cost, explicit | 47% |
| "cost of muffins & buns" | 21% |
| 4 muffins, 3 buns | 9% |
| total amount, implied | 7% |
| number of muffins & buns | 5% |

23. **If the equation $(x + 1)^3 + x = 349$ is true when x = 6, then what value of x will make the following equation true?**

$$(5x + 1)^3 + 5x = 349$$

| | |
|---|---|
| x=6/5, 1 1/5, 1.2 | 28% |
| x=30 | 6% |
| $1 < x < 2$ | 2% |
| uncodable | 31% |
| no response | 30% |

24. **Blue pencils cost 5 cents each and red pencils cost 6 cents each. I buy some blue and some red pencils and altogether it costs me 90 cents.**

**If b is the number of blue pencils bought, and if r is the number of red pencils bought, what can you write down about b and r?**

| | |
|---|---|
| 5b+6r=90 | 55% |
| b+r=90 | 20% |
| b=12, r=5 | 3% |
| b=6, r=10 | 2% |
| 2 or more correct pairs | 2% |
| incorrect pair | 3% |